# Inductive Logic review

Recall Hume: We have observed the sun rise every day our whole lives. How are we justified in believing it will rise again tomorrow?

Hume (and many other logicians) proposes a dichotomy between

- Deduction
  - Certain
  - Mathematical
  - Conclusions contain no more information than the premises
- Induction
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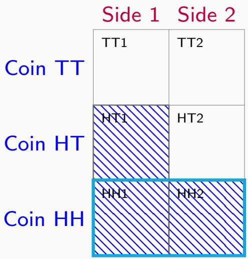
At least two factions emerge in response to Hume:

- Falsificationists (e.g. Popper 1963): We do not need inductive logic
- Bayesians (e.g. Carnap 1966): Bayesian probability formalizes inductive logic

Today I will describe a fascinating skirmish in the war between the falsificationsists and Bayesians: the Popper-Miller theorem.

# Running Example




We observe heads = $HT1 \vee HH1 \vee HH2$

We chose the HH coin = $HH1 \vee HH2$

Recall that the truth $\times$ lies in exactly one cell.

Here is a running example for this presentation. Suppose a bag contains three coins: one regular coin (HT), one with both faces tails (TT), and one with both faces heads (HH). The coin is flipped, and either the first or second side comes up.

Exactly one possible outcome of coin $\times$ side occurs; call this the "truth."

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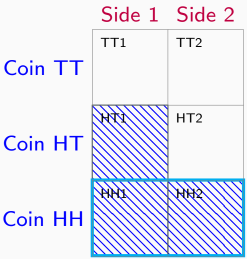


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Recall logical notation:

- $\vee$: Disjunction (logical or)
- $\wedge$: Conjunction (logical and)
- $\neg$: Negation (logical not)

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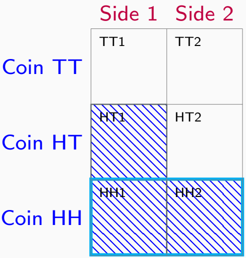


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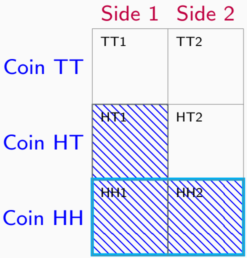
Let us define a "hypothesis" $h$ and evidence $e$:

- Hypothesis $h$: We selected the HH coin (all future flips will be H)
- Evidence $e$: We observed heads.

How much does $e$ support $h$? This is an inductive question! Call the answer $s(h|e)$.

## Running Example

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Bayes' rule gives a potential answer:

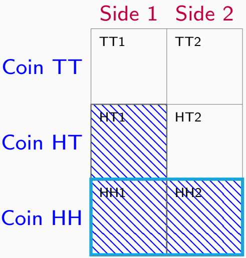
$$s(h|e) = p(h|e) - p(h).$$

In this case,

$$s(h|e) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} > 0.$$

This is positive, so we might say that $e$ supports $h$. Have we solved induction?

# The Popper-Miller theorem

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The first step of their argument is to break $h$ into deductive and inductive components.

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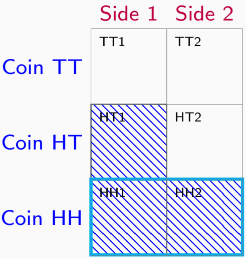


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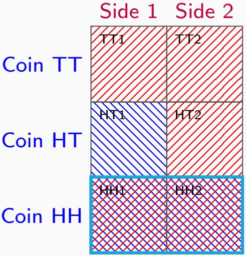
1. $e$ follows deductively from $h$, since $h \subseteq e$.
2. But $h$ does not follow deductively from $e$, because $e \not\subseteq h$.
3. Does some "component" of $h$ follow from $e$?
4. The strongest proposition implied by $e$ and containing $h$ is $h_D := h \vee e$.
5. Can we write $h = h_D \wedge S$ for some set?
6. The weakest such propostion is $h_I := h \vee \neg e$.

So we can write $h = h_D \wedge h_I$. Popper and Miller call:

- $h_D$ : The "deductive" component of $h$
- $h_I$ : The "inductive" component of $h$




- $e$ = We observe heads
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- $h_D$ = Inductive component = $h \vee e = e$
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We can write $h = h_D \wedge h_I$.

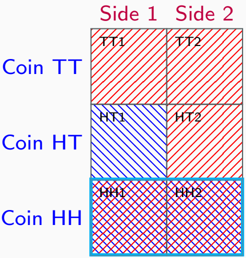
Note also that $h \subseteq e$ so $h_D = e$.

What is the support for these two components? Basic calculations show that

$$\begin{aligned} s(h_D|e) &= & p(h_D|e) - p(h_D) &= & 1 - p(e) \\ s(h_I|e) &= & p(h_I|e) - p(h_I) &= & p(h|e) - (p(h) + 1 - p(e)) \\ \Rightarrow s(h|e) &= & p(h|e) - p(h) &= & s(h_D|e) + s(h_I|e) \end{aligned}$$

So the support of $h$ is the sum of the support for its components.

# The Popper-Miller theorem

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So the support of $h$ is the sum of the support for its components. Next, by Bayes' rule,

$$s(h|e) = \frac{p(h \wedge e)}{p(e)} - p(h) = \frac{p(h)}{p(e)} - p(h) = \frac{p(h)}{p(e)}(1 - p(e)) < 1 - p(e) = s(h_D|e).$$

So $h$ is supported less than its deterministic component. Also sensible.

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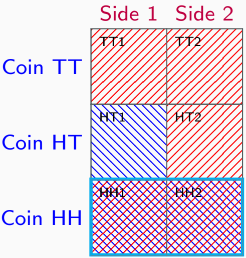


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So $h$ is supported less than its deterministic component. Also sensible. But then:

$$s(h_I|e) = s(h|e) - s(h_D|e) < 0.$$

$\Rightarrow$ **The inductive component is counter-supported by the evidence.**

# The Popper-Miller theorem

To summarize Popper-Miller's argument:

1. We can write $h = h_D \wedge h_I$ where $h_D$ follows deductively from $e$.
2. We have chosen $h_I$ so that $s(h|e) = s(h_D|e) + s(h_I|e)$.
3. It is reasonable to call $h_D$ and $h_I$ the deductive and inductive components of $h$.
4. But $s(h_D|e) > s(h|e)$, so $s(h_I|e) < 0$.

That is, $h$ is only supported by $e$ because it depends partly, deductively, on $e$.

Any non-deductive dependence of $h$ on $e$ is actually *counter-supported* by the evidence.

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> "This result is completely devastating to the inductive interpretation of the calculus of probability. All probabilistic support is purely deductive: that part of a hypothesis that is not deductively entailed by the evidence is always strongly countersupported by the evidence — the more strongly the more the evidence asserts."
> (Popper and Miller 1983)

# The discussion continues

This idea generated a lot of discussion, and a detailed follow-up by Popper and Miller. Some notable references:

- Levi and Jeffrey 1984
- Redhead 1985
- Levi 1986
- Good 1990
- Popper and Miller 1987

To me, this result has the feel of a paradox. Though maybe not very practically relevant, by taking it seriously one is forced to think very carefully and potentially identify unarticulated assumptions or unjustified conculsions.

**What do you think?**

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